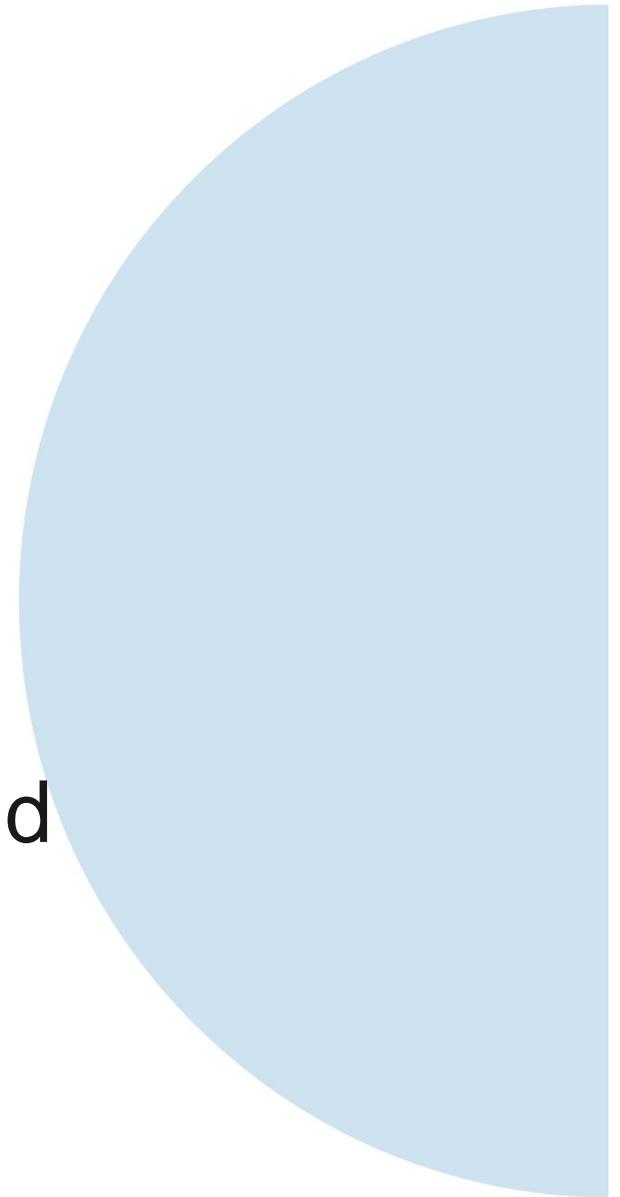# The combinatorial clock auction format

Operation of
the supplementary bid round and
the second price rule

3 November 2009

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Et komplekst auktionsdesign?

- Følgende forudsætninger skal opfyldes:
  - Antallet af tilladelser ikke fastlagt
  - Frekvensmængden pr. tilladelse ikke fastlagt
  - Sammenhængende spektrum i hver tilladelse
  - Opnå frekvenser efter individuelle forretningsplaner

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Fokusere på to elementer i CCA

- Supplerende budrunde
- "Second price" reglen

The supplementary round

WHY IT IS NECESSARY

The CCA is an open auction

- The primary rounds and the supplementary rounds together form an open auction

  - Price revelation over a number of rounds

- To keep it simple, bidders are restricted to bid for only one package in each primary round

  - Activity rules encourage bidders to bid for their most preferred package in each primary round

  - We may not have enough information at the end of the primary rounds to decide the most efficient outcome…

  - …and bidders may not have made bids for alternative acceptable outcomes

The supplementary round

- The supplementary round is not a stand-alone, sealed bid:
    - Offers a chance to make bids for other packages
    - Express true valuation of the various packages
    - Reduces risk for bidders of being disappointed with the outcome
- Prices are determined by what bidders need to pay to win, not what they actually bid (more about this later).

The relative cap on supplementary bids

- Bids in the supplementary round are capped according to choices in the primary round which eliminates "surprises" in the supplementary bids round

- Bidders are limited on how much *extra* they can bid for packages larger than they bid for in the final primary round

- Limits the ability of bidders to disrupt the outcome after the final primary round

- There are **simple strategies** for winning your position as it stood after the final primary round

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The supplementary round

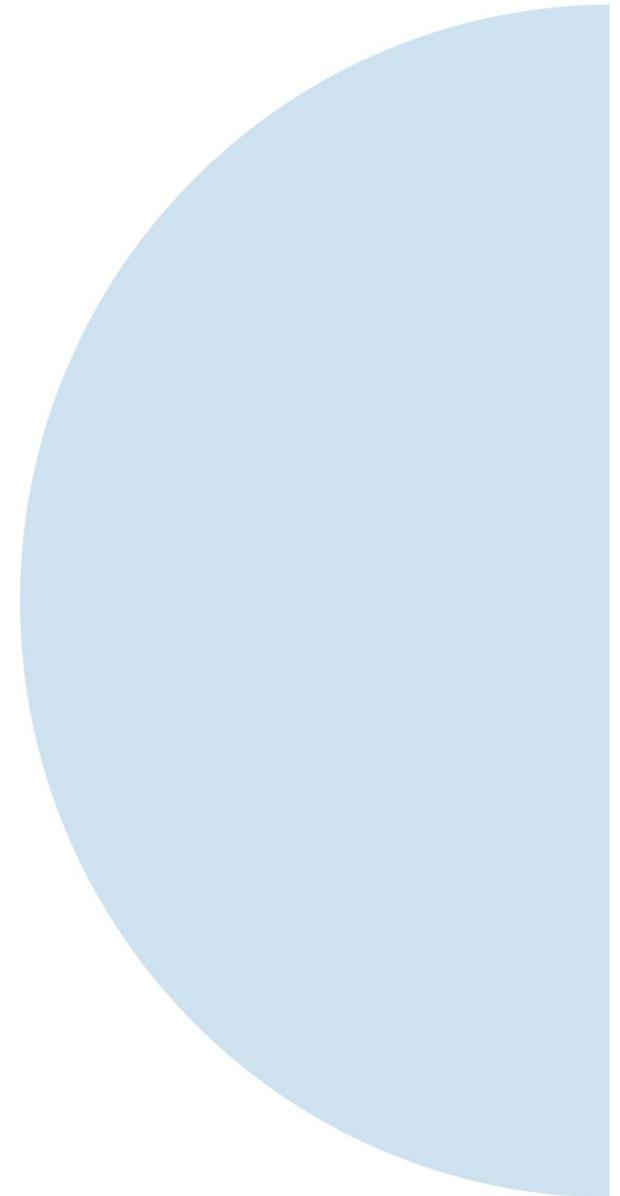# SOME EXAMPLES
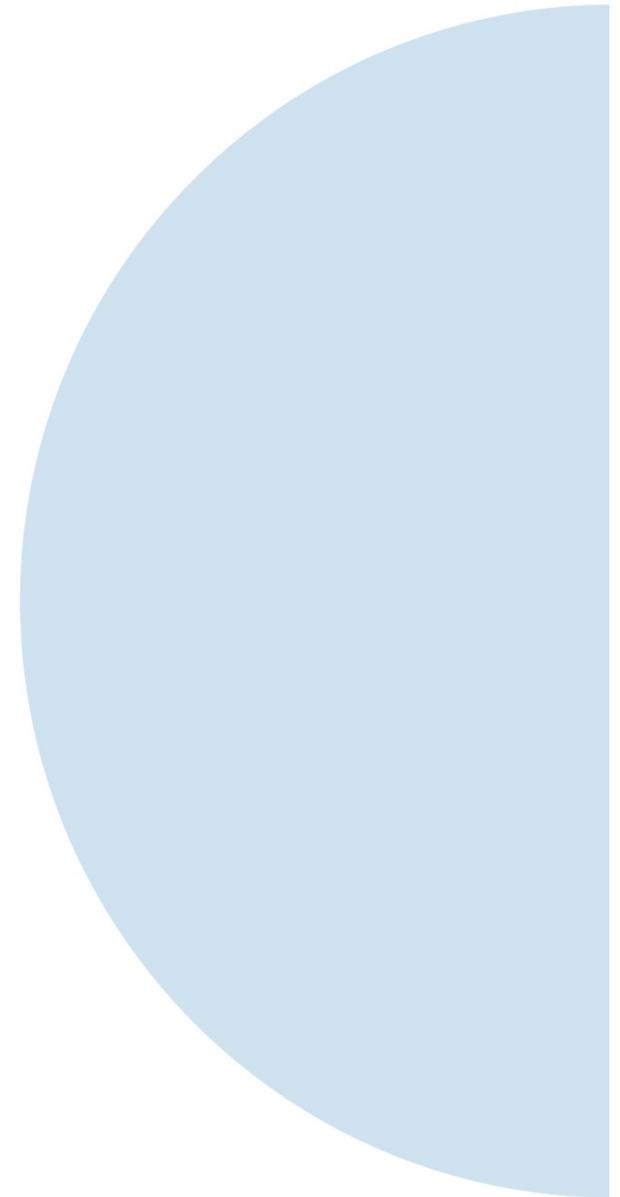
Example 1

- There are 5 bidders competing for 14 generic lots with the following valuations:

Bidder	Number of lots	Valuations (DKK)
Adam	**4**	**12.1m**
Adam	**2**	**5m**
Benjamin	4	10m
Carl	4	10m
David	4	6.3m
Emilie	**4**	**6m**
Emilie	**2**	**3.2m**

If there were no supplementary round…

- It is the 5th primary round and the price per lot is DKK1.4m

- At this price, all 5 bidders would bid for 4 lots as the cost of this package (DKK5.6m) gives more or same surplus as 2 lots

- There is excess demand of 6 lots in this round and the auctioneer raises the price per lot to DKK2m in the 6th round

- At DKK2m per lot, Adam, Benjamin and Carl will still bid for 4 lots

- However, David and Emilie will not bid for any lots as the price exceeds their valuations

- No excess demand in 6th primary round and if there was no supplementary round, the auction would end with 2 lots going unsold

- **This outcome is clearly inefficient when there is losing bidder (Emilie) who values 2 lots above the reserve price**

If there were a supplementary round…

- All bidders could express their true valuation of a package of 4 lots as they know they will only pay what they need to win, not what they actually bid
    - Adam – DKK12.1m
    - Benjamin and Carl – each bid DKK10m.
    - David – DKK6.3m
    - Emilie – DKK6m
- Adam and Emilie have the opportunity to submit a bid for 2 lots:
    - Adam - DKK5m
    - Emilie - DKK3.2m
- The winner determination process will allocate 4 lots each to Adam, Benjamin and Carl (same outcome for them as close of primary rounds) and 2 lots to Emilie – No unsold lots!
- This is a frequency efficient outcome

Example 2

- Again, suppose it is the 5th primary round and the price per lot is DKK1.4m

- All 5 bidders have the same valuations as before and all 5 bidders place a bid for 4 lots

- There is excess demand of 6 lots and the auctioneer increases the price per lot in the next round to *DKK1.6m (rather than DKK2m in previous example)*

- In this round, Emilie and David can no longer afford to bid for 4 lots. David drops demand to zero while Emilie bids for 2 lots at *DKK3.2m (her valuation)*

- There is no excess demand and the primary stage ends

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In the supplementary round…

- Bidders can express their true valuation for their final primary bid:
    - For 4 lots:
        - Adam – DKK12.1m
        - Benjamin and Carl – DKK10m.
        - David – DKK6.3m
        - Emilie – DKK6m
    - For 2 lots:
        - Adam – DKK5m

- Emilie does not place a supplementary bid for 2 lots in this case as she already bid her valuation for 2 lots in the final primary round.

- In this case, the supplementary round process makes no material difference to the auction outcome as the primary round allowed the bidders to express their willingness to pay for their preferred package

Example 3

- Again, suppose the price per lot was DKK1.4m in the 5th primary round and 5 bidders (Adam, Benjamin, Carl, David and *Erika*) bid for 4 lots

- The price per lot was set at DKK2m in the 6th primary round. Only Adam, Benjamin and Carl bid for 4 lots

- In the final primary round, at DKK2m, there were 2 unsold lots

- Because Erika and David dropped demand to zero in the 6th round, their relative cap in the supplementary round will be determined by the price in the 6th primary round:

Bidder	Number of lots	Cap (DKK)
David	4	8m
Erika	4	8m
Erika	2	4m

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Add supplementary round bids

- Assume the bidders had the following valuations and made supplementary bids so that the overall bid summary is:

Bidder	Number of lots	Bid (DKK). Primary bid (PB in round n) or Supp. Bid (SB)	Valuations (DKK)
Adam	4	9m (SB)	12.1m
Adam	2	5m (SB)	5m
Benjamin	4	8m (PB6th)	10m
Carl	4	8m (PB6th)	10m
David	4	5.6 (PB5th)	6.3m
Erika	4	7.7m (SB)	7.7m
Erika	2	3.6m (SB)	3.6m

- The winning combination of bids that will maximise total winning bids is: 2 lots for Adam and 4 lots for Benjamin, Carl and Erika

- Adam does not win 4 lots as his bid for 2 lots offer a more efficient outcome given other bids.

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Winning the final primary bid package

- If Adam wanted to make sure he won 4 lots, he could have bid the sum of his final primary round bid (DKK8m) plus the value of the unsold lots (2 at DKK2m each) in the final primary round plus DKK1000 (to avoid ties) = DKK12.001m whilst not bidding for 2 lots.

- A sole bid of DKK12.001m for 4 lots would have ensured that Adam would win regardless of other bids

- Erika and David had a cap on their suppl. bids (2 lots, max DKK4m and 4 lots max DKK8m)

- Neither Erika nor David could place a supplementary bid high enough to stop Adam winning 4 lots if Adam had increased his bid by the value of the unsold lots in final primary round and refrained from bidding for other packages.

Bidder	Number of lots	Bid (DKK). Primary bid (PB in round n) or Suppl. Bid (SB)	Valuations (DKK)
Adam	4	9m (SB)	12.1m
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Erika	4	7.7m (SB)	7.7m
Erika	2	3.6m (SB)	3.6m

Base prices for winning bids

Bidder	Winning Package	Base Price (DKK)
Adam	2	1.5m
Benjamin	4	5.6m
Carl	4	5.6m
Erika	4	5.6m

The following section takes a further look at how prices are calculated.

Note: In the above example the opportunity cost of Adam winning 2 lots is DKK1.5m. That is the value of denying David from winning 4 lots (DKK5.6m) and Erika from winning 2 lots (DKK3.6m), a total of DKK9.2m, minus Erika's value of 4 lots (DKK7.7m) which she will win if Adam won 2 lots.

The opportunity cost of Benjamin, Carl and Erika winning 4 lots is denying David from winning 4 lots which he values at DKK5.6m. Therefore the base price of Benjamin, Carl and Erika's winning package of 4 lots is DKK5.6m.

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The second price rule

HOW IT WORKS
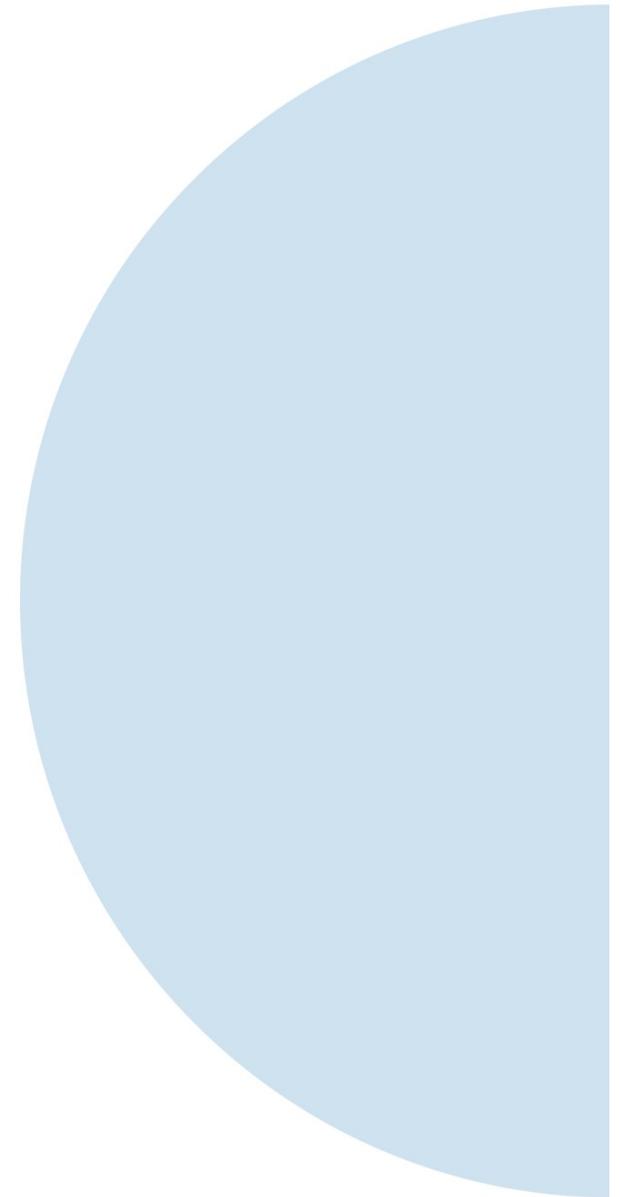
The second price rule

- The winning combination of bids is the combination of compatible bids that maximises total value

- However, winning bidders typically do not pay the amount of their winning bid

- Winners only pay the opportunity costs, i.e. the cost of denying the next highest bidders access

- In effect, bidders only pay the least amount that they could have bid and still won

The second price rule

SOME EXAMPLES

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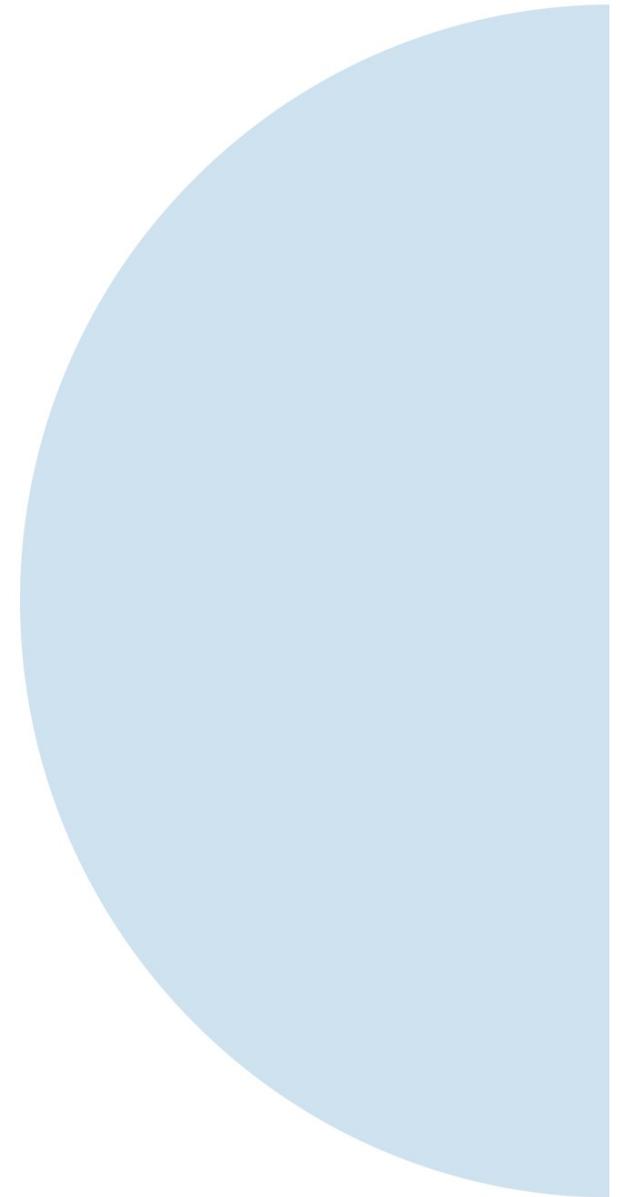
Example 4

- Imagine an auction with just one lot for sale and three bidders:
    - Karina highest bid is 21
    - Karen highest bid is 26
    - Lene highest bid is 19
- Karen wins (as she bid the highest value) but pays only 21 (the opportunity cost of denying Karina the lot)
- If bidders bid their value they will either win or be 'happy losers':
    - If you win, you pay your valuation or less, meaning you have surplus
    - If you lose, the price paid by the winner is your valuation or above and at this price you would prefer to lose
- The second price rule in the auction is more complicated because of packaging bids, but the basic idea that bidders only pay what they need to win, not what they bid, is still true

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Example 5

First primary round

Demand for 2.5GHz unpaired is 18 lots > supply of 9(+1)

Demand for 2.5GHz paired is 20 lots > supply of 14

A second primary round is announced

Bidder	2010-2025MHz	2.5GHz unpaired lots of 5MHz	2.5GHz paired lots of 2x5MHz	Bid amount (DKK, mill.)
Anna			4	4
Beatrice			4	4
Carla			4	4
Daniel			4	4
Erik			4	4
Frederik		9		4.5
Gertrud		9		4.5

Second primary round

Demand for 2.5GHz paired is 19 lots > supply of 14.

A third primary round is announced

Bidder	2010-2025MHz	2.5GHz unpaired lots of 5MHz	2.5GHz paired lots of 2x5MHz	Bid amount (DKK, mill.)
Anna			4	*8*
Beatrice			4	*8*
Carla			4	*8*
Daniel			4	*8*
Erik			*3*	6
Frederik		*9*		9
Gertrud		*0*		*0*

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Third primary round

Becomes the last primary round as demand is less than or equal to supply for all categories.

Bidder	2010-2025MHz	2.5GHz unpaired lots of 5MHz	2.5GHz paired lots of 2x5MHz	Bid amount (DKK, mill.)
Anna			*0*	*0*
Beatrice			4	*10*
Carla			4	*10*
Daniel			4	*10*
Erik			*2*	*5*
Frederik		9		9
Gertrud				

Supplementary round

To keep things simple, suppose no bidder submits more than one supplementary bid.

Bidder1	2010-2025MHz	2.5GHz unpaired lots of 5MHz	2.5GHz paired lots of 2x5MHz	Bid amount (DKK, mill.)
Anna				
Beatrice			4	31
Carla			4	35
Daniel			4	32
Erik			2	7
Frederik			4	4
Gertrud				

Winner determination

The maximum value combination is worth DKK114 million.

The four bidders who were bidding for paired spectrum in the last primary round win paired spectrum accordingly.

The bidder who bid for unpaired spectrum in the last primary round wins unpaired.
BUT:

Bidder	2010-2025MHz	2.5GHz unpaired lots of 5MHz	2.5GHz paired lots of 2x5MHz	Bid amount (DKK, mill.)
Beatrice			4	31
Carla			4	35
Daniel			4	32
Erik			2	7
Frederik		9		9
Total	0	9(+1) lots	14 lots	DKK114m

Pricing

Beatrice, Carla and Daniel each pay only DKK 8 million – equal to Anna's highest bid for 2x20MHz in the second round and the opportunity cost of denying her bid.

Erik pays DKK2million – the reserve price.

Frederik pays DKK4.5million – equal to Gertrud's highest bid in the first round and hence the opportunity cost of denying her bid.

Bidder	2010-2025MHz	2.5GHz unpaired lots of 5MHz	2.5GHz paired lots of 2x5MHz	Bid amount (DKK, mill.)
Beatrice			4	8
Carla			4	8
Daniel			4	8
Erik			2	2
Frederik		9		4.5
Total	0	9(+1) lots	14 lots	DKK30.5m

VARIATION – Winner determination

Assume two of the bidders have much higher valuations than other bidders and bid accordingly.

The maximum value combination is now worth DKK187 million.

BUT:

Bidder	2010-2025MHz	2.5GHz unpaired lots of 5MHz	2.5GHz paired lots of 2x5MHz	Bid amount (DKK, mill.)
Beatrice			4	31
Carla			4	*60*
Daniel			4	*80*
Erik			2	7
Frederik		9		9
Total	0	9(+1) lots	14 lots	DKK187m

Pricing

The opportunity cost of denying Anna 2x20MHz is still DKK8 million so the prices to be paid remain the same.

Bidder	2010-2025MHz	2.5GHz unpaired lots of 5MHz	2.5GHz paired lots of 2x5MHz	Bid amount (DKK, mill.)
Beatrice			4	8
Carla			4	8
Daniel			4	8
Erik			2	2
Frederik		9		4.5
Total	0	9(+1) lots	14 lots	DKK30.5m

Thank you!

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